[5 PTS] Solve the inequality $\frac{1}{x-3} \le \frac{9}{4x+3}$. Write your answer in interval notation. **ANSWER:** $\left(-\frac{3}{4}, 3\right) \cup [6, \infty)$

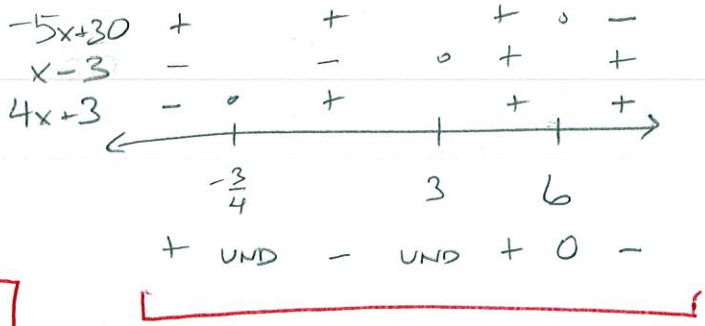
$$\frac{1}{x-3} - \frac{9}{4x+3} \le 0$$

$$\frac{4x+3-9(x-3)}{(x-3)(4x+3)} \le 0$$

$$\frac{-5x+30}{(x-3)(4x+3)} \le 0$$

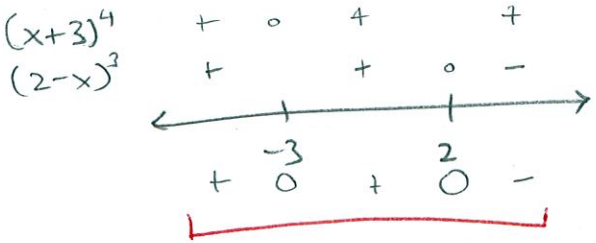
$-5x+30=0 \Rightarrow x=6$

$(x-3)(4x+3)=0 \Rightarrow x=3, -\frac{3}{4}$



[3 PTS] Solve the inequality $(x+3)^4(2-x)^3 > 0$. Write your answer in interval notation. **ANSWER:** $(-\infty, -3) \cup (-3, 2)$

$(x+3)^4(2-x)^3=0 \Rightarrow x=-3, 2$



ADDITIONAL QUESTIONS ON THE OTHER SIDE ➡

[12 PTS] Let $f(x) = \dfrac{x^3}{x^2-4}$. $= \dfrac{x^3}{(x+2)(x-2)}$

[a] Find the domain of f. Write your answer in interval notation.

ANSWER: $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$ (1½)

$x^2-4 \neq 0$

$x \neq \pm 2$

[b] Find the intercepts of f.

ANSWER: (½) x – intercept 0

(½) y – intercept 0

$f(0) = \dfrac{0}{-4} = 0$ $\qquad \dfrac{x^3}{x^2-4} = 0$

$x^3 = 0 \rightarrow x = 0$

[c] Find all vertical asymptotes of f, if they exist.
If there are no vertical asymptotes, write DO NOT EXIST and skip to part [e].

ANSWER: $x = \pm 2$

[d] **Skip this part if the correct answer to [c] is DO NOT EXIST.**
Following the process shown in lecture,
find the behavior on each side of each vertical asymptote.

ANSWER: (½) As $x \rightarrow 2^+$, $f(x) \rightarrow \infty$

(½) As $x \rightarrow 2^-$, $f(x) \rightarrow -\infty$

(½) As $x \rightarrow -2^+$, $f(x) \rightarrow \infty$

(½) As $x \rightarrow -2^-$, $f(x) \rightarrow -\infty$

As $x \rightarrow 2^+$, $f(x) \approx \dfrac{8}{4(x-2)} = \dfrac{2}{x-2}$ ← >0 IF $x>2$, <0 IF $x<2$

$x \rightarrow 2^-$,

$x \rightarrow -2^+$, $f(x) \approx \dfrac{-8}{-4(x+2)} = \dfrac{2}{x+2}$ ← >0 IF $x>-2$, <0 IF $x<-2$

$x \rightarrow -2^-$,

[e] Find all horizontal asymptotes of f, if they exist.
If there are no horizontal asymptotes, write DO NOT EXIST.

ANSWER: DO NOT EXIST

[f] Find all slant asymptotes of f, if they exist.
If there are no slant asymptotes, write DO NOT EXIST.

ANSWER: $y = x$

$x^2-4 \overline{)\,x^3}$ $\quad x$

$x^3 - 4x$

$4x$

[g] **Skip this part if the correct answers to [e] and [f] are DO NOT EXIST.**
Find the x – coordinates of all points where the graph of f intersects its horizontal and/or slant asymptotes.

ANSWER: $x = 0$

$\dfrac{x^3}{x^2-4} = x$ WHEN REMAINDER $\dfrac{4x}{x^2-4} = 0$

@ $x = 0$

[h] Sketch the graph of f on the grid shown to the right.

ANSWER: (2½)